ON THE STRUCTURE OF SHOCK WAVES

(O STRUKTURE UDARNYKH VOLN)

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It is well known that the equations of gasdynamics and magnetohydrodynamics admit discontinuous solutions if the coefficients of viscosity and thermal conductivity are assumed to be zero ($\eta = 0$, $\chi = 0$), and the electrical conductivity is assumed to be infinitely large ($\sigma = \infty$). The discontinuities in these solutions satisfy definite algebraic relations.

On the other hand, for more exactly formulated equations, the discontinuous solutions are replaced by continuous ones. If such a continuous solution tends to a discontinuous one as $\eta \to 0$, $\chi \to 0$ and $\sigma \to \infty$, then it is called a shock wave. For sufficiently small values of η, χ, and σ^{-1}, the shock wave may be replaced by a jump discontinuity. However, not every jump discontinuity satisfying the indicated algebraic relations is necessarily the limit of a shock wave. Therefore, neglecting the dissipative coefficients η, χ, and σ^{-1}, we shall consider only those discontinuous solutions which are the limits of continuous solutions. The jumps in such discontinuous solutions will be termed admissible.

In order to distinguish admissible jumps from inadmissible ones, two methods exist at the present time. In the first method, the admissibility of a given jump is established after proving the existence of the corresponding shock wave [1,2], or even after calculating the shock wave [3-6]. The second method [7-11] is based on the fact that certain jump discontinuities, when subjected to infinitesimal disturbances, split into several propagating jump discontinuities. Such (non-evolutionary) jumps, which are unstable with respect to splitting, will be considered inadmissible. All other jumps will be assumed admissible*. From the

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* Let $V_1^- \leqslant V_2^- \leqslant \ldots \leqslant V_n^-$ be the phase velocities of small disturbances in the region left of the shock front, and $V_1^+ \leqslant V_2^+ \leqslant \ldots \leqslant V_n^+$ be the same quantities to the right. Let $n_-(n_+)$ denote the number of phase

actual solutions of some problems (cf., for example, [12-16]), it is shown that these remaining jumps are sufficient for the Cauchy problem to possess a unique solution.

So far, there exists only fragmentary evidence that the two points of view indicated above must yield the same result [1,2].

In the present paper, this question is considered in connection with the so-called dissipative system of equations, of which the ordinary and magnetohydrodynamic equations are special cases. It will be shown that the condition of stability (with respect to splitting) is a necessary condition in order that the jump correspond to a unique shock wave (within translation).

We shall also clarify in which cases the profile of the shock wave contains jumps. This phenomenon was discovered by Marshall [4] and was studied in Whitham's paper [5], with which the present work has many points in agreement.

1. Dissipative systems. Let us consider the system of quasi-linear equations of the form

$$\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x} A_j(u) = \sigma_j \psi_j(u) \qquad (j = 1, \ldots, n) \tag{1.1}$$

where $A_j(u) \equiv A_j(u_1, u_2, \ldots, u_n)$ and $\psi_j(u)$ are differentiable functions of the same arguments; moreover, $\psi_j(u) \equiv 0$ for $j \leqslant m$ $(m < n)$.

The equality $u = u^\circ$ (u° being a constant vector) defines a constant and uniform solution of system (1.1) if $\psi_j(u^\circ) = 0$ $(j = m+1, \ldots, n)$. The set of all such vectors u° will be denoted by M.

Starting from the system (1.1), let us consider the system obtained from (1.1) by linearizing around the point $u = u^\circ \in M$

$$\frac{\partial v_j}{\partial t} + \sum_{k=1}^{n} A_{jk}(u^\circ) \frac{\partial v_k}{\partial x} = \sigma_j \sum_{k=1}^{n} \psi_{jk}(u^\circ)\, v_k, \qquad v_j = u_j - u_j^\circ \tag{1.2}$$

where

$$A_{jk}(u^\circ) = \frac{\partial A_j(u^\circ)}{\partial u_k^\circ}, \qquad \psi_{jk}(u^\circ) = \frac{\partial \psi_j(u^\circ)}{\partial u_k^\circ}$$

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velocities V_j^- (V_j^+) smaller than (greater than) the shock speed U. For the jump to be stable with respect to splitting, it is necessary that $n_- + n_+ = n - 1$.

The system (1.1) will be called dissipative if for any arbitrary $u^\circ \in M$ the following conditions hold:

1) In system (1.2) all particular solutions of the form

$$v_j(x, t) = a_j e^{i(\omega t - kx)} \qquad (-\infty < k < \infty)$$

are bounded for $t > 0$, for whatever non-negative numbers $\sigma_j (j = m + 1, \ldots, n)$.

2) If all the numbers $\sigma_j (j = m + 1, \ldots, n)$ are positive and finite, then these particular solutions tend to zero for $t \to \infty$ (except the solution with $k = 0$, $\omega = 0$).

In what follows, we shall consider the system (1.1) to be dissipative. This implies, as is easily seen, that none of the roots $\omega = \omega_s$ $(s = 1, 2, \ldots, n)$ of the equation

$$D(\omega, k) \equiv \det | i\omega\delta_{js} - ikA_{js}(u^\circ) - \sigma_j\psi_{js}(u^\circ) | = 0 \tag{1.3}$$

lies in the lower half-plane; moreover, if all the coefficients $\sigma_j (j = m + 1, \ldots, n)$ are positive and finite, then the real axis is free from these roots also.

2. Shock waves. We shall be interested in those solutions of system (1.1) which are shock waves moving with some constant velocity U without changes in their forms. These solutions depend only on the variable $\xi = x - Ut$, they satisfy the system of ordinary differential equations

$$-U\frac{du_j}{d\xi} + \frac{d}{d\xi}A_j(u) = \sigma_j\psi_j(u) \tag{2.1}$$

and they tend to some limits $u^+ \in M$ and $u^- \in M$ as $\xi \to \infty$. Furthermore, $\lim du/d\xi = 0$ as $\xi \to \pm\infty$. For brevity we shall call these solutions transitional solutions.

Evidently the vectors u^- and u^+ are connected by the relations

$$-Uu_j^- + A_j(u^-) = -Uu_j^+ + A_j(u^+) \qquad (j = 1, \ldots, m)$$

$$\psi_j(u^-) = \psi_j(u^+) = 0 \qquad (j = m + 1, \ldots, n)$$

These are the conditions which must be satisfied by the jump $u^+ - u^-$.

Let us obtain a simple condition necessary for the existence of a transitional solution. For sufficiently large absolute values of $\xi (\xi < 0)$, system (2.1) may be linearized into

$$-U(u_j - u_j^-) + \sum_{s=1}^{n} A_{js}(u^-)(u_s - u_s^-) = 0 \qquad (j = 1, \ldots, m) \tag{2.2}$$

$$-U\frac{du_j}{d\xi} + \sum_{s=1}^{n} A_{js}(u^-)\frac{du_s}{d\xi} = \sigma_j \sum_{s=1}^{n} \psi_{js}(u^-)(u_s^- - u_s^-) \qquad (j = m+1, \ldots, n)$$

The linearized system has particular solutions of the form

$$u^{(r)} - u^- = a^{(r)} \exp \nu_r^- \xi \tag{2.3}$$

where ν_r^- $(r = 1, 2, \ldots, n - m)$ are the roots of the equation

$$D_1(\nu, U) \equiv \det \begin{vmatrix} -U\delta_{js} + A_{js}(u^\circ) \\ (-U\delta_{js} + A_{js}(u^\circ))\nu - \sigma_j\psi_{js}(u^\circ) \end{vmatrix} = 0, \qquad u^\circ = u^- \tag{2.4}$$

If $\mathrm{Re}\,\nu_r^- > 0$, then the difference $u^{(r)} - u^-$ tends to zero as $\xi \to -\infty$. Obviously the transitional solution $u(\xi)$ in the domain considered may be represented as a linear combination thus:

$$u - u^- = \sum_{r=1}^{\rho^-} C_r^- a^{(r)} \exp \nu_r^- \xi \tag{2.5}$$

where the summation is extended over all values of r for which $\mathrm{Re}\,\nu_r^- > 0$. In a similar way, in the domain of large positive ξ the function $u(\xi)$ may be represented thus:

$$u - u^+ = \sum_{r=1}^{\rho^+} C_r^+ b^{(r)} \exp \nu_r^+ \xi \tag{2.6}$$

where the summation is extended over all r with $\mathrm{Re}\,\nu_r^+ > 0$.

The solutions (2.5) and (2.6) obtained in this fashion may be extended, at least in principle, to the point $\xi = 0$ with the aid of the exact equations (2.1). At this point, both solutions must have identical components. Moreover, one of the components, say u_1, may be required to assume the prescribed value $u_1(0)$ $(u_1(0) \in (u_1^-, u_1^+))$; this is permissible, because if there exists one transitional solution $u(\xi)$, $(u(-\infty) = u^-$, $u(+\infty) = u^+)$, then there must exist an infinite set of such solutions, namely, $u(\xi - a)$ $(-\infty < a < \infty)$. By choosing the parameter a we may require $u_1(0)$ to be any prescribed value in the interval (u_1^-, u_1^+).

Thus there are $n + 1)$ conditions to be satisfied at $\xi = 0$:

$$u_1(-0) = u_1(0), \qquad u_s(-0) = u_s(+0) \qquad (s = 1, \ldots, n)$$

To satisfy these conditions we have at our disposal ρ^+ parameters C_r^+ and ρ^- parameters C_r^-. If the number of these parameters $(\rho^+ + \rho^-)$ is less than the number of conditions $(n+1)$, then a continuous transitional solution, generally speaking, cannot be constructed. If $\rho^{+}+\rho^{-}>n+1$, then evidently there exists an entire family of different transitional solutions with the same component $u_1(0)$. Therefore for the existence of a unique transitional solution (except for translation), we must have

$$\rho^{-}+\rho^{+}=n+1 \tag{2.7}$$

Our aim is to establish the connection between these conditions and the $n^{-}+n^{+}=n-1$ conditions of stability relative to splitting (cf. footnote, p. 1559). To solve this purely algebraic problem, we shall utilize some specific properties of the dissipative systems.

3. Ideal systems. Instead of the system (1.1) let us consider a sequence of auxiliary systems obtained in the following manner. We set some of the coefficients σ_j to zero and the remaining to infinity. We number the components u_j such that $\sigma_j=0$ for $j=1, 2, \ldots, m_1 \geqslant m$ and $\sigma_j=\infty$ for $j=m_1+1, \ldots, n$. System (1.1) assumes the form

$$\begin{aligned} &\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x}A_j(u)=0 \qquad (j=1,\ldots,m_1)\\ &\psi_j(u)=0 \qquad (j=m_1+1,\ldots n)\end{aligned} \tag{3.1}$$

The number m_1 $(m \leqslant m_1 \leqslant n)$ may be called the rank of system (3.1). Evidently there exists one system of rank m or n, $(n-m)$ systems of rank $m+1$ or $n-1$, etc.

Corresponding to the system (3.1), the dispersion equation (1.3) assumes the form

$$\Delta_{m_1}(i\omega, ik) \equiv \det\left|\begin{matrix} i\omega\delta_{js}-ikA_{js}(u^\circ)\\ \psi_{js}(u^\circ)\end{matrix}\right|=0 \tag{3.2}$$

We easily see that

$$\Delta_{m_1}(i\omega, ik)=(ik)^{m_1}\Delta_{m_1}\left(\frac{\omega}{k}, 1\right)\equiv(ik)^{m_1}\Delta_{m_1}(V), \qquad V=\frac{\omega}{k} \tag{3.3}$$

Thus the phase velocity $V=\omega/k$ corresponding to the system (3.1) satisfies the equation $\Delta_{m_1}(V)=0$ and does not depend on k. Moreover, it is always real; since, if it were not the case, as k varies, ω would pass from the upper half-plane to the lower half-plane (or vice versa), and this is inconsistent with a dissipative system (1.1). A real phase velocity means that a plane wave of the form $u=a\exp[i(\omega t-kx)]$ will be undamped. Therefore, all systems of type (3.1) are ideal systems. If some of the dissipative coefficients are small and the remaining coefficients very large, then to a high degree of accuracy we may replace the original system (1.1) with the corresponding system (3.1). Thus the system (3.1) is of definite interest.

Let us consider a system (C), which is obtained from the system (3.1) by replacing one of the equations $\psi_\alpha(u) = 0$ ($\alpha = m_1 + 1, \ldots, n$) with the equation

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x} A_\alpha(u) = 0$$

We shall call this system of equations an adjacent system.

The following theorem holds.

Theorem (Whitham [5]). The system (3.1) of rank m_1 has exactly m_1 phase velocities $V_1 \leqslant V_2 \leqslant \ldots \leqslant V_{m_1}$ each velocity being counted the same number of times as the multiplicity of the root in the equation $\Delta_{m_1}(V) = 0$. The phase velocities $V_1' \leqslant V_2' \leqslant \ldots \leqslant V_{m_1+1}'$ of an arbitrary adjacent system will alternate with the phase velocities of the system (3.1) thus:

$$V_1' \leqslant V_1 \leqslant V_2' \leqslant V_2 \leqslant \ldots \leqslant V_{m'} \leqslant V'_{m'+1}$$

Proof. In system (1.1) we set

$$\sigma_{m+1} = \sigma_{m+2} = \ldots = \sigma_{m_1} = 0, \quad 0 < \sigma_{m_1+1} < \infty, \quad \sigma_{m_1+2} = \ldots = \sigma_n = \infty$$

Equation (1.3) thereby assumes the following form:

$$\det \begin{vmatrix} i\omega\delta_{js} - ikA_{js}(u^\circ) \\ i\omega\delta_{m_1+1s} - ikA_{m_1+1s} - \sigma_{m_1+1}\psi_{m_1+1s}(u^\circ) \\ \psi_{js}(u^\circ) \end{vmatrix} = 0$$

which may be written as

$$\Delta_{m_1+1}(i\omega, ik) - \sigma_{m_1+1}\Delta_{m_1}(i\omega, ik) = 0$$

or, using (3.3), we may yet rewrite it as

$$w(V) \equiv \Delta_{m_1+1}(V) + i\,\frac{\sigma_{m_1-1}}{k}\,\Delta_{m_1}(V) = 0 \tag{3.4}$$

Let us consider this dispersion relation for positive values of k. Since the system (1.1) is dissipative, all the roots V of Equation (3.4) lie in the upper half-plane or on the real axis.

From this, according to the theorem of Hermite and Biler, we immediately conclude the alternation of the phase velocities. Without citing the formulation of this theorem, we reproduce the essential steps of the proof. To begin with, we assume that the polynomials $\Delta_{m_1+1}(V)$ and

$\Delta_{m_1}(V)$ have no zeros in common; consequently, all the zeros of polynomial $w(V)$ have positive imaginary parts. When the point V runs along the entire real axis from left to right, the point w describes some curve in the complex plane; this curve does not pass through the origin, and the argument of the point w increases monotonically. The point w alternately crosses the real and the imaginary axes. Therefore, the zeros of its real and imaginary parts alternate. This proof also remains valid when the functions Δ_{m_1+1} and Δ_{m_1} have one or more common zeros.

From the alternation of the zeros of the polynomials $\Delta_{m_1+1}(V)$ and $\Delta_{m_1}(V)$, it follows that the orders of the two polynomials cannot differ by more than one. The order of the polynomial $\Delta_n(V)$ is evidently equal to n, while that of $\Delta_{n-1}(V)$ cannot exceed $n - 1$ and thus must equal $n - 1$. Extending this argument, we see that the order of the polynomial $\Delta_{m_1}(V)$ equals $m_1 (m_1 = m, \ldots, n)$. The theorem is thus proved.

We make a further observation. Let a_{m_1+1} denote the coefficient of the highest term in polynomials $w(V)$ and $\Delta_{m_1+1}(V)$. The point $w(V)/a_{m_1+1}$ moves in such a way that its argument increases with increasing V. As V tends to $+\infty$, the argument of $w(V)/a_{m_1+1}$ tends to zero, and consequently, for sufficiently large V, it is negative. This shows that

$$\operatorname{Im} \frac{1}{a_{m_1+1}} w(V) = \sigma_{m_1+1} \frac{\Delta_{m_1}(V)}{k a_{m_1+1}} < 0$$

for sufficiently large V. From this it follows that the coefficient a_{m_1} of the highest term of the polynomial $\Delta_{m_1}(V)$ has a sign opposite to that of a_{m_1+1}.

Considering this fact, and recalling the alternation of the phase velocities of the adjacent systems, we conclude that

$$\begin{aligned} &\Delta'_{m_1+1}(V_j')\, \Delta_{m_1}(V_j') \leqslant 0 \qquad (j = 1, \ldots, m_1 + 1) \\ &\Delta_{m_1+1}(V_j)\, \Delta'_{m_1}(V_j) \geqslant 0 \qquad (j = 1, \ldots, m_1) \end{aligned} \tag{3.5}$$

4. Motion of the root ν of Equation (2.4). We shall clarify how the root ν of Equation (2.4) moves in the complex plane as the parameter U moves along the real axis.

From the definitions of the functions $D(\omega, k)$ and $D_1(\nu, u)$, the following relation results:

$$D(i\nu U, -i\nu) \equiv \nu^m D_1(\nu, U) \tag{4.1}$$

If we assume that for any arbitrary $U(-\infty < U < \infty)$ one of the roots ν

of Equation (2.4) is purely imaginary ($\nu = ik_0 \neq 0$), then we conclude that the equation $D(\omega, k_0) = 0$ possesses a real solution $\omega = -k_0U$. Since this is impossible for a dissipative system, then as U moves along the real axis none of the roots ν of Equation (2.4) may cross the imaginary axis at any point, except $\nu = 0$.

We shall clarify for which values of U one of the roots may vanish. From the definition (2.4) of the function $D_1(\nu, U)$, it follows that

$$D_1(0, U) \equiv \Delta_m(U) \prod_{j=m+1}^{n} (-\sigma_j) \tag{4.2}$$

Therefore, the point $\nu = 0$ is a root of the equation $D_1(\nu, U) = 0$ when and only when U coincides with one of the phase velocities

$$V_1^\circ \leqslant V_2^\circ \leqslant \ldots \leqslant V_m^\circ \tag{4.3}$$

of the system (C°) of the lowest rank m.

Let us consider in greater detail that root $\nu(U)$ of Equation (2.4), which vanishes for $U = V_\alpha^\circ$ (α being one of the numbers 1, 2, ..., m).

To begin with, we assume that V_α° is not the phase velocity of at least one of the systems (C) of rank $m + 1$. Then $U = V_\alpha^\circ$ is a simple zero of the function $\Delta_m(U)$, and consequently

$$\frac{\partial}{\partial U} D_1(0, U)\Big|_{U=V_\alpha^\circ} = (-1)^n \Delta_m'(V_\alpha^\circ) \prod_{j=m+1}^{n} \sigma_j \neq 0$$

On the other hand, it follows from the form of the function $D_1(\nu, U)$ that

$$\frac{\partial}{\partial \nu} D_1(\nu, V_\alpha^\circ)\Big|_{\nu=0} = (-1)^n \Big(\prod_{j=m+1}^{n} \sigma_j\Big) \sum_{s=m+1}^{n} \frac{\Delta_{m+1, s}(V_\alpha^\circ)}{\sigma_s}$$

Here $\Delta_{m+1, s}$ ($s = m + 1, \ldots, n$) are the determinants corresponding to all the possible systems (C) of rank $m + 1$.

From the inequalities (3.5) it follows that none of the terms in the last sum may possess a sign opposite to that of $\Delta_m'(V_\alpha^\circ)$. Since by our assumption these terms cannot all vanish, then the derivatives

$$\frac{\partial}{\partial U} D_1(0, U)\big|_{U=V_\alpha^\circ}, \qquad \frac{\partial}{\partial \nu} D_1(\nu, V_\alpha^\circ)\big|_{\nu=0}$$

have the same sign. Thus $\nu'(V_\alpha^\circ) < 0$. It is possible to show that this derivation remains valid if V_α° is the phase velocity of all systems of rank $m + 1$.

Therefore each time the parameter U (monotonically increasing) crosses one of the phase velocities (4.3) of the system (C°), one of the zeros ν of the equation $D_1(\nu, U) = 0$ crosses over from the right half-plane to the left half-plane along the real axis.

It does not follow, however, that when the parameter V moves in an interval not containing any phase velocity V_a° ($a = 1, \ldots, m$), then the number of roots ν of the equation $D_1(\nu, U) = 0$ remains constant in each half-plane.

In fact, a root $\nu(U)$ may pass from one half-plane to the other through infinity without crossing the imaginary axis.

Such a passage actually occurs each time the coefficient of the highest term in ν in the polynomial $D_1(\nu, U)$ vanishes. This coefficient, evidently, equals $(-1)^n \Delta_n(U)$. It vanishes when $U = V_j^*(j = 1, \ldots, n)$, V_j^* being the phase velocities of the systems (C) of rank n.

Using reasoning similar to that above, one easily shows that each time the monotonically-increasing parameter U crosses one of the phase velocities $V_j^*(j = 1, \ldots, n)$ of the systems (C^*) of the highest rank, one of the zeros ν of the equation $D_1(\nu, U) = 0$ crosses from the left half-plane to the right, tending to infinity when $U = V_j^*$. In this manner, the number of roots $\nu(U)$ lying on one or the other side of the imaginary axis changes only when the parameter U crosses a phase velocity of the system (C°) or (C^*), i.e. system of the highest or lowest rank.

Let us now fix the parameter U and vary the point $u^\circ \in M$, thus changing the phase velocities $V_j^\circ = V_j^\circ(u^\circ)$ ($j = 1, \ldots, m$) and $V_j^* = V_j^*(u^\circ)$ ($j = 1, \ldots, n$). If in this process none of the phase velocities intersects the quantity U, then the number $l(U, u^\circ)$ of roots $\nu(U)$ in the left half-plane will not change. Thus, the number

$$l\,(-\infty) = \lim\, l\,(U, u^\circ)$$

as $U \to -\infty$ is the same for all $u^\circ \in M$.

The three conclusions drawn at this point may be unified by the following formula:

$$l\,(U,\, u^\circ) = l\,(-\infty) + n^\circ\,(U,\, u^\circ) - n^*\,(U,\, u^\circ) \tag{4.4}$$

where $n^\circ(U, u^\circ)$ and $n^*(U, u^\circ)$ are the number of phase velocities $V_j^\circ(u^\circ)$ and $V_j^*(u^\circ)$, respectively, smaller than U. Using this relation, we may calculate the sum $\rho^- + \rho^+$ (cf. Section 2) knowing only the relative positions of the points $V_j^\circ(u^-)$, $V_j^\circ(u^+)$, $V_j^*(u^-)$, $V_j^*(u^+)$, and U on the real axis. In fact, it follows immediately from (4.4) that

$$\rho^- + \rho^+ = n + \delta n^\circ - \delta n^* \tag{4.5}$$

where

$$\delta n^\circ = n^\circ (U, u^+) - n^\circ (U, u^-), \qquad \delta n^* = n^* (U, u^+) - n^* (U, u^-)$$

5. Continuous profile of a shock wave and profile with a jump. We shall clarify how to determine from the relative positions of the phase velocities V_j° and V_j^* and the shock speed U on the real line whether a transitional solution exists, whether it is unique (within translation), and whether it is continuous.

Let us assume that some transitional solution $u(x)$ exists.

Let us first consider the simplest case, in which each of the differences $V_j^* - U$ $(j = 1, \ldots, n,\ V_j^*(x) \equiv V_j^*[u(x)])$ have the same sign at all points $-\infty < x < \infty$. Then $\delta n^* = 0$, and by virtue of (4.5) the stability condition of the jump (relative to splitting) $\delta n^\circ = 1$ implies condition (2.7) $\rho^- + \rho^+ = n + 1$. Therefore, in this case, to an evolutionary wave there corresponds a unique continuous profile (within translation), and to a non-evolutionary wave there corresponds either no transitional solution ($\delta n^\circ < 1$) or an infinite set of them ($\delta n^\circ > 1$).

We now let some of the differences $V_j^*(x) - U$ change sign. Let $x = a$ be the point, in the neighborhood of which one of the differences changes sign, decreasing monotonically. The point a cannot be a point of discontinuity, since such a discontinuity would be non-evolutionary. Thus, the functions $u_j(x)$ $(j = 1, \ldots, n)$ are continuous at this point.

It is possible to show that at the point $x = a$ there either coincide at least two of the phase velocities of the system (C*), or that all of the derivatives $du/d\xi$ are finite. In either case, at the point $x = a$ besides the condition of continuity we must satisfy one further condition: the condition of solvability of system (2.1) with respect to the derivative $du/d\xi$ (remembering that the determinant of the system vanishes at $x = a$), or the condition of two phase velocities coinciding (assuming for simplicity that the phase velocities do not coincide identically). Thus, the presence of p points of this type implies the existence of p additional conditions.

Let us now consider the point $x = b$, in the neighborhood of which the difference $V_j^* - U$ changes sign, monotonically increasing. At this point an evolutionary discontinuity may exist. We observe that the location of the point $x = b$ may vary within known limits. Therefore, the presence of q points of this type implies the existence of q additional free parameters. Condition (2.7), valid in the absence of points of type a or b, may be rewritten as

$$\rho^+ + \rho^- + q = n + 1 + p \tag{5.1}$$

Taking into account that $\delta n^* = q - p$, we write the last relation with the aid of (4.5) in the form

$$\delta n^\circ = 1 \tag{5.2}$$

Thus, the necessary condition for the existence of a unique shock wave corresponding to a given discontinuity will be the evolutionary condition (5.2).

If, in addition, $\delta n^* > 0$, then the profile of the wave will contain discontinuities, the number of which will be not less than δn^*.

In conclusion, we observe without proof that all the eigenvalues of the matrix

$$\left\| \sigma_j \frac{\partial \psi_j(u)}{\partial u_k} \right\|_{m+1}^{n}, \quad u \in M, \quad 0 < \sigma_\alpha < \infty, \quad \alpha = m+1, \ldots, n$$

are positive if the system (1.1) is fully dissipative.

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